

Influence of Currents on Equilibrium Range Spectra of Wind Waves

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Abstract

It is well known that water waves tend to be higher or lower as they propagate onto an opposing or following current, respectively. This is also true for wind waves generated on currents. In the equilibrium range of wind waves, however, the spectral densities for the waves generated on a following or opposing current are larger and smaller, respectively, than those for the waves generated on quiescent water. To see this, a series of laboratory experiments was carried out in a wind-wave and current flume for various conditions of water depth, current and wind. The experimental results qualitatively confirm the theoretical equations proposed by Gadzhiyev et al. in 1978 and Suh et al. in 1994 with the former performing marginally better for following currents and the latter doing for opposing currents. It is also shown that the partially developed laboratory waves fall under gravity waves so that the laboratory regime represents the open ocean situation.

Key words: currents, equilibrium, interactions, laboratory tests, water waves, wave spectra, wind waves

INTRODUCTION

Wave-current interaction has long been a subject of interest in the areas of

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coastal engineering and physical oceanography. There are two different situations in which water waves interact with currents. The first is the occasion on which waves generated on quiescent water propagate into a current region. An example is that an offshore-generated swell propagates towards shore and encounters strong currents at a river mouth or a tidal inlet. The second case is wind waves being generated on flowing water. An example is the evolution of wind waves in a shallow basin where tidal currents are relatively strong (e.g., development of northwesterners in the Yellow Sea between China and Korea).

For the first situation, Huang et al. (1972) first derived an equation that describes the influence of currents on the change of a wave spectrum in deep water. This equation was extended to finite-depth water by Hedges et al. (1985). However, these equations did not take into account the enhanced level of wave breaking induced by opposing currents, especially associated with the equilibrium range of the spectrum. Hedges (1981) modified Huang et al.'s (1972) theoretical model in deep water to allow for the limit on spectral densities brought about by wave breaking, and a similar modification to Hedges et al.'s (1985) model was made by Suh et al. (1994) for waves in finite-depth water.

For the second situation, laboratory experiments were carried out by Francis and Dudgeon (1967) and Kato and Tsuruya (1978). In both of these studies, they found that when the wind is opposing the flow the waves tend to be higher than when it follows the flow. In the equilibrium range (or the high-frequency portion) of wind wave spectra, however, spectral densities of the waves generated on a following current could be higher than those of the waves generated on quiescent water or on an opposing current. In the equilibrium range, direct energy input from wind and nonlinear energy transfer from the lower frequencies are balanced by energy dissipation due to wave breaking. In deep water, waves break if the wave steepness (or the ratio of wave height to wavelength) exceeds a certain value. It is well known that the wavelength becomes longer on a following current than on quiescent water for a given wave frequency, and the reverse is true on an opposing current. Therefore, when the wind follows the flow, more wind energy can be absorbed in the waves without breaking than when the wind blows over quiescent water. Consequently, we can hypothesize that in the equilibrium range the spectral densities can be larger as waves are generated on a following current than when they are generated on quiescent water. On the other hand, when the waves are generated on an opposing current, it is expected that there will be an equilibrium range spectrum of less energy density than on quiescent water.

Equations for predicting the equilibrium range spectrum of waves generated on a following or opposing current have been proposed by Kitaigorodskii et al. (1975) and Hedges (1981) for deep water and by Gadzhiyev et al. (1978) and Suh et al. (1994) for finite-depth water. Hedges et al. (1985) and Suh et al. (1994) also carried out laboratory experiments for opposing currents in paddle-generating wave flumes and showed that in the equilibrium range both theoretical and experimental spectral densities on opposing currents are less than those on quiescent water. For a following current, however, usual paddle-generating wave makers cannot be used to see the enhancing energy densities (compared to quiescent water) in the equilibrium range because no wind energy is supplied to the waves and thus energy densities over the entire frequency range always decrease as the waves encounter a following current. Instead, a wind-wave and current flume should be used to see the influence of following currents on the equilibrium range spectra.

In this paper, we report laboratory experiments made in a wind-wave and current flume to examine the equilibrium range spectra of wind waves generated on following or opposing currents. Comparison is also made with the theoretical results of Gadzhiyev et al. (1978) and Suh et al. (1994). Because these theories have been developed for a vertically uniform current, the equivalent uniform current proposed by Hedges and Lee (1992) is used to take into account the effect of the actual depth-varying shear current.

EXPERIMENT

Experimental Apparatus

Experiments were carried out in the wind-wave and current flume at the Hydrodynamics Laboratory of the Japan Port and Harbour Research Institute. Diagrams of the flume are shown in Figs. 1 and 2. The uniform test section is 1.5 m wide, 1.3 m high and 28.5 m long. The side walls consist of glass plates and the top of the flume is covered with wood plates. On the windward (right hand) side of the test section over the waterway is a wind blower, which generates wind by an axial fan driven by a 50 kW variable-speed motor. The wind then passes through guide vanes, a fine mesh screen and honeycombs so that the wind velocity at the inlet section (see Fig. 1) is quite uniform. At the inlet, a horizontal guide plate is provided, which can be adjusted vertically so as to be located at the water surface.

Currents can be generated by pumping the water through the pipe as shown in Fig. 1. The direction of the current can be altered by the operation of the valves. The current velocity can be controlled by adjusting the flow rate accurately by means of a venturi meter.

Wind velocity, current velocity and water surface displacement were measured with an anemometer, electromagnetic current meters and resistance-type wave gauges, respectively, all manufactured by the KENEK Electronics Company. The LABTECH NOTEBOOK data acquisition software was used, which directly stores the data in EXCEL files in the personal computer while displaying the data on the monitor screen in real-time during the measurement.

Experimental Procedure and Data Analysis

First, currents were generated, while the wind fan was not in operation, and current velocities were measured at nine elevations dividing the total water depth by 10 equidistant intervals at St. B and D in Fig. 2. At each elevation, the current velocity was measured for 60 s at a sampling rate of 20 Hz to obtain the time-averaged current velocity. The two velocities obtained at the same elevations at St. B and D were then averaged to obtain a single representative current velocity in the flume. The vertical current velocity profile, $u(z)$, was determined using the equation proposed by Coleman (1981):

$$u(z) = u_{\max} + u_* Z \quad (1)$$

with

$$Z = \frac{1}{K} \ln\left(\frac{z}{d}\right) - \frac{2P}{K} \left[1 - \sin\left(\frac{\pi z}{2d}\right) \right] \quad (2)$$

in which u_{\max} = maximum current velocity at the free surface; u_* = current shear velocity; K = von Karman constant (= 0.4 for clear water); P = a constant related to the turbidity of fluid (= 0.19 for clear water); z = vertical distance measured upward from the bed surface; and d = water depth. The quantities of u_{\max} and u_* were calculated from the linear regression analysis of the measured current velocities on Z . The thus calculated u_{\max} and u_* are given in Table 1. The measured current profiles are presented in Fig. 3 for different

water depths along with the profiles curve-fitted by (1).

Second, wind was generated over quiescent water and wind velocities were measured at nine elevations at an increment of 5 cm (i.e., at 5, 10, ..., 45 cm) above the still water level at St. C in Fig. 2. When the wave height was so large that the water elevation reached the anemometer at 5 cm, the measurement at 5 cm was omitted. At each elevation, measurement was made for 60 s at a sampling rate of 20 Hz to obtain the time-averaged wind velocity. The vertical wind velocity profile, $v(y)$, was determined using the equation as follows:

$$v(y) = v_{\max} + \frac{v_*}{K} \ln\left(\frac{y}{h}\right) \quad (3)$$

in which v_{\max} = maximum wind velocity at 45 cm above the still water level; v_* = wind shear velocity; y = vertical distance measured upward from the still water level; and h = the highest elevation of wind measurement (45 cm in this experiment). The quantities of v_{\max} and v_* were calculated from the linear regression analysis of the measured wind velocities on $\ln(y/h)/K$. The thus calculated v_{\max} and v_* are given in Table 1. An example of the measured and curve-fitted wind profiles is shown in Fig. 4. The wind profiles are not used in this study except to convert wind velocities to a wind shear velocity, which is sometimes used to represent the strength of wind.

In this experiment, the wind velocity profile was measured at only one location, approximately the center of the flume. The roughness of the water surface and the corresponding shear velocity and vertical wind velocity profile may vary along the flume. However, because the fetch is not so long, their variation may not be significant. Actually, Kato and Tsuruya (1978) had made a similar experiment in the same flume to find that the shear velocity did not vary significantly along the flume. However, they found that in general the shear velocity was larger in the cases of opposing current than in the cases of following current, reflecting the roughness of the water surface.

Third, with only the wind blowing, wave measurements were made at the four locations, A, B, D and E, as shown in Fig. 2. To permit the slower-traveling high-frequency component waves to travel to the remote wave gauge at St. E, a sufficient waiting time was allowed to elapse after the initiation of wind generation prior to data acquisition. A total of 36,000 data points was collected at the sampling rate of 50 Hz for each of the four wave gauges.

Fourth, with both wind and current, wave measurements were made at the four locations, A, B, D and E, as shown in Fig. 2. The methods of data acquisition are the same as those for the no-current case.

In the spectral analysis of the wave data, the smoothing techniques presented in Otnes and Enochson (1978) were used. Of 36,000 data points in each test record, the first 20,480 data points were processed in nine segments of 4,096 points per segment. These segments overlap by 50% for smoother and statistically more significant spectral estimates. The time series of each segment was corrected by applying a 10% cosine taper on both ends and was subjected to spectral analysis. The raw spectra were then ensemble-averaged. Further smoothing was made by band-averaging over five neighboring frequency bands. The total number of degrees of freedom is 60 for the final spectra.

The above procedure was repeated for different conditions of water depth, current and wind. Experiments were made for three different water depths; 50, 40 and 30 cm. For the water depths of 50 and 40 cm, three different velocities of following and opposing currents were tested, and for each current velocity three different wind velocities were tested. For the water depth of 30 cm, two different velocities of following and opposing currents were tested, and for each current velocity two different wind velocities were tested. Of course, for each water depth, wave measurements were also made in quiescent water for different wind velocities. The test conditions and the calculated parameters for currents and winds are given in Table 1.

THEORY

By combining the theoretical expressions originally developed by Kitaigordskii et al. (1975), Gadzhiyev et al. (1978) proposed an equation for the equilibrium range of a wave spectrum in the presence of currents in water of finite depth (see also Massel (1996), pp. 269-271):

$$S_K^e(\omega_a, d, U) = \alpha g^2 \omega_a^{-5} (1 + 3p\omega_a R) \Phi_K \quad (4)$$

in which the superscript e denotes the equilibrium range spectrum, the subscript K refers to Kitaigordskii et al., ω_a = absolute angular frequency in the stationary frame of reference; U = vertically uniform current velocity in the

direction of wave propagation (i.e., positive for following current); α = Phillips' constant; g = gravitational acceleration; $p = 1$ or -1 for unidirectional waves propagating on following or opposing currents, respectively; $\omega_u = \omega_a U / g$; and

$$\Phi_K(\omega_a, d) = \kappa^{-2} \left[1 + \frac{2\omega_d^2 \kappa}{\sinh(2\omega_d^2 \kappa)} \right]^{-1} \quad (5)$$

$$R(\omega_a, d) = \frac{2\kappa}{1 + \kappa^2 \omega_d^2 - \omega_a^2} + \frac{1}{3} \left[\frac{1}{\kappa^3 \Phi_K} - 2\kappa \Phi_K (1 - 2\omega_d^2 \kappa^2 + \kappa^2) - 4\omega_d^2 \kappa \right] \quad (6)$$

In the preceding equations, $\omega_d = \omega_a (d / g)^{1/2}$, and the function $\kappa(\omega_d)$ is determined as the solution of the transcendental algebraic equation

$$\kappa \tanh(\omega_d^2 \kappa) = 1 \quad (7)$$

In (4), Φ_K represents the effect of finite water depth and the parenthesized term on the right-hand side is for the effect of currents.

On the other hand, by extending the equation developed by Hedges (1981) for a deep-water spectrum, Suh et al. (1994) proposed an equation for the equilibrium range spectrum of waves propagating on currents in finite-depth water as follows:

$$S_S^e(\omega_a, d, U) = \frac{1}{D} \alpha g^2 \omega_r^{-5} \Phi_K(\omega_r, d) \quad (8)$$

in which the subscript S refers to Suh et al., $\Phi_K(\omega_r, d)$ is calculated by (5) and (7) with $\omega_d = \omega_r (d / g)^{1/2}$,

$$D = 1 + \frac{\omega_r U}{g} \left\{ \frac{2}{\sqrt{\tanh\left(\frac{\omega_r^2}{g} d\right)}} - \frac{\frac{\omega_r^2}{g} d}{\cosh\left(\frac{\omega_r^2}{g} d\right) \left[\tanh\left(\frac{\omega_r^2}{g} d\right) \right]^{3/2}} \right\} \quad (9)$$

and ω_r = relative angular frequency in the frame of reference moving with the current, which is related to ω_a by

$$\omega_r = \omega_a - kU \quad (10)$$

Here k is the wave number which is calculated from the dispersion relationship

$$\omega_r^2 = (\omega_a - kU)^2 = gk \tanh(kd) \quad (11)$$

The foregoing theories for wave-current interaction were developed based on the assumption of a vertically uniform current, but the actual currents generated in the experiment were depth-varying shear currents. Hedges and Lee (1992) introduced the so-called equivalent uniform current, U_e , defined as the uniform current which produces the same wavelength, L , as the actual depth-varying current for a given observed wave period and water depth. The equivalent uniform current, U_e , is given by

$$U_e = \frac{1}{\varepsilon L} \int_{d-\varepsilon L}^d u(z) dz \quad (12)$$

in which εL is given by

$$\varepsilon L = \frac{\tanh kd}{k} \quad (13)$$

The curve-fitted current velocity profile given by (1) was used in (12) to calculate the equivalent uniform current. In the following analysis, the equivalent uniform current, U_e , was used in place of U in (4) and (8). Note that for a higher frequency εL becomes smaller so that the equivalent uniform current becomes

larger.

RESULTS

In the experiment, wave measurements were made at the four locations, A, B, D and E, as shown in Fig. 2. The waves at St. A close to the wind inlet, however, behaved almost like capillary waves, having a spectral peak period of 0.14 to 0.27 s. Therefore these data were not included in the following analysis. Such spectral parameters as peak frequency and peak spectral density were required in the following analysis. For determining these parameters from the measured wave spectra, the method proposed by Günther (1981) and summarized in the paper of Aranuvachapun (1987) was used.

The wind wave spectrum for partially developed waves on quiescent water of finite depth is expressed as

$$S_o(\omega, d) = S_p \Phi_J \Phi_K(\omega, d) \quad (14)$$

with

$$S_p(\omega) = \alpha g^2 \omega^{-5} \quad (15)$$

$$\Phi_J(\omega, \omega_p, \gamma, \sigma) = \exp \left\{ -1.25 \left(\frac{\omega_p}{\omega} \right)^4 + \ln \gamma \right\} \exp \left[- \frac{(\omega - \omega_p)^2}{2\sigma^2 \omega_p^2} \right] \quad (16)$$

in which the subscript o refers to the quantities in quiescent water, ω = wave angular frequency; ω_p = peak angular frequency; S_p = Phillips' equilibrium range spectrum in deep water; Φ_J = JONSWAP shape function with a peak enhancement factor γ and a spectral width parameter σ ; and $\Phi_K(\omega, d)$ given by (5) with ω in place of ω_a is the Kitaigorodskii shape function representing the effect of finite water depth.

In the equilibrium range (or the high-frequency portion) of the spectrum, the JONSWAP shape function has no influence. The equilibrium range spectrum on quiescent water of finite depth is therefore given by

$$S_o^e(\omega, d) = \alpha g^2 \omega^{-5} \Phi_K(\omega, d) \quad (17)$$

The choice of the equilibrium range is somewhat arbitrary. In this study it was taken as the frequency range of $1.35\omega_p \leq \omega \leq 3\omega_p$. On the other hand, the equilibrium range spectrum of the waves generated on currents in finite-depth water can be expressed as

$$S^e(\omega_a, d, U) = \beta g^2 \omega_a^{-5} \Phi_K(\omega_a, d) \quad (18)$$

in which β = a Phillips-like constant for the waves generated on currents. As discussed in the introduction, it is expected that β will be greater than α for the waves generated on a following current and the reverse is true for the waves generated on an opposing current.

There are slightly different methods for determining α from the measured spectrum. The standard method is to take the mean of $S_o g^{-2} \omega^5 \Phi_J^{-1} \Phi_K^{-1}(\omega, d)$ in the equilibrium range (i.e., $1.35\omega_p \leq \omega \leq 3\omega_p$), where the JONSWAP shape function has no influence. Theoretically, α must be a constant but α determined by the above-mentioned method varies slightly depending upon water depth, wind velocity and locations of wave measurement. In this experiment, the higher the wind velocity, the larger the value of α , and in general, α decreased with the distance from the wind inlet. Omitting the data at St. A, α varied between 0.0206 and 0.0832. In the following analysis, α was taken as the average of the α 's calculated at the three locations, B, D and E, and the averaged α is given in Table 1.

The final goal of this study is to compare β/α between experiment and theory. For the experimental data, β is calculated by

$$\beta = S_m g^{-2} \omega_a^5 \Phi_K^{-1}(\omega_a, d) \quad (19)$$

in which S_m = measured spectral density. The thus calculated β becomes large

near the spectral peak, where the effect of the JONSWAP shape function is significant for the measured spectrum. For the theories of Gadzhiyev et al. (1978) and Suh et al. (1994), β is calculated by replacing S_m in (19) by S_K^e in (4) or S_S^e in (8), respectively.

Figs. 5 to 7 show the comparison of β/α between experiment and theories for a following current in different water depths. Only the data at St. E are presented. As expected, both experiment and theories show β/α greater than unity in the equilibrium range ($1.35 \leq \omega_a/\omega_{a_p} \leq 3.0$), which increases with frequency. The effect of water depth is not significant. The theoretical result of Gadzhiyev et al. (1978) gives a better agreement with the experimental result than that of Suh et al. (1994) which gives a slight overestimation. In the experiment, higher harmonics in the vicinity of ω_a/ω_{a_p} of 2.0 and 3.0 due to triad coupling (Young and Eldeberky, 1998) are observed, which cannot be predicted by the present theoretical models.

To see the effect of current velocity, the results for different current velocities are presented in Figs. 8 and 9, which should be compared with Fig. 6 having the same water depth and wind velocity. Comparing Fig. 8 with Fig. 6, it is observed that β/α decreases with decreasing current velocity as expected. The rate of increase with frequency is also diminished. Fig. 9, compared with Fig. 6, shows that both the magnitude and rate of increase with frequency of β/α increase with the current velocity.

To see the effect of wind velocity, the results for different wind velocities are presented in Figs. 10 and 11, which should be compared with Fig. 6 having the same water depth and current velocity. It is observed that both the magnitude and rate of increase with frequency of β/α decreases with increasing wind velocity, probably because α increases with wind velocity as stated previously.

Finally, the predictability of the theoretical equations is examined for an opposing current, even though it has previously been tested for the data obtained in paddle-generating wave flumes by Hedges et al. (1985) and Suh et al. (1994). The solution to (11) does not exist for high-frequency wave components on a strong opposing current. Therefore test cases were selected for which the current was the weakest and the wind was the strongest. As the wind becomes strong, the spectrum

is shifted towards lower frequencies. Figs. 12 and 13 show the results for the waves generated on the weakest opposing current by the strongest wind in different water depths. As expected, both experiment and theories show β/α less than unity in the equilibrium range. The result of Gadzhiyev et al. (1978) decreases monotonically with frequency. The result of Suh et al. (1994), however, decreases with frequency but at very high frequencies bounds up to the point where the solution to (11) does not exist. This effect is caused by the fact that D in (8) approaches zero in this high frequency range. In this laboratory experiment, the fetch was so short that the overall frequency was very high. For sufficiently developed wind waves in the field, this kind of problem may not occur.

In the foregoing analysis, β is not a constant but varies with frequency. Recalling that β is defined as a Phillips-like constant for the waves generated on currents and that α was determined by taking the mean of $S_o g^{-2} \omega^5 \Phi_J^{-1} \Phi_K^{-1}(\omega, d)$ in the equilibrium range, it may be necessary to examine the value of β averaged over the equilibrium range. The thus averaged β is now a constant like α , and thus β/α is also a constant. As shown in Figs. 12 and 13, for some cases the theoretical values of β/α could not be calculated for very high-frequency component waves. In these cases, both theoretical and experimental values of β were averaged in the range from $1.35\omega_{a_p}$ up to the highest frequency for which the theoretical value could be calculated. The thus calculated values of β/α are presented in Table 2. Like the results shown in Figs. 5 to 13, β/α is greater and smaller than unity for following and opposing currents, respectively. For following currents, both theories systematically give larger values of β/α than the experiment and Gadzhiyev et al.'s (1978) theory gives better agreement with the measurements than Suh et al.'s (1994) one. For opposing currents, Suh et al.'s (1994) theory is in better agreement with the experiment than Gadzhiyev et al.'s (1978) one.

The waves in this laboratory experiment are only partially developed so that both wave period and wave length are short compared with those of real ocean gravity waves. The readers may have a question whether the laboratory regime represents the open ocean situation. All the results so far are shown in terms of the nondimensional frequency, ω_a/ω_{a_p} , and there is no mention of what the frequencies are in dimensional terms. To give an idea of this question, the peak

frequency f_{a_p} at St. E is given in Table 1 for each test. The wave spectrum at St.

E is the most developed one in the experiment and was used for comparison with theories. As expected, the peak frequency decreases with increasing wind speed and for the same wind it becomes larger and smaller on following and opposing current, respectively, compared with that on quiescent water. It does not change much with the water depth. The largest one is 4.084 Hz at the strongest following current (F3) and the weakest wind (W1) in the water depth of 40 cm. The upper bound of the equilibrium range in this case is 12.25 Hz, which is close to the borderline between gravity waves and capillary waves. Note that the equilibrium range was taken as the frequency range from $1.35f_{a_p}$ to $3f_{a_p}$ in this study.

Therefore, the waves in the equilibrium range fall under gravity waves in almost all the cases. Fig. 14 shows the evolution of the wave spectrum with the fetch for the above case. For the shortest fetch (St. B) the equilibrium range falls under capillary waves, but as the fetch increases it moves towards the gravity wave region. The secondary peak appeared at $f_a \approx 1.0$ Hz may be due to a characteristic of the wave flume. Such a peak always appears near the frequency of 1.0 Hz without regard to the experimental conditions though its magnitude varies.

CONCLUSIONS

In order to examine the influence of currents on the equilibrium range spectrum of wind waves, a series of laboratory experiments has been performed for various water depths, wind velocities and current velocities in a wind-wave and current flume. The experimental results have been compared with the theoretical equations proposed by Gadzhiyev et al. (1978) and Suh et al. (1994). A good agreement between experiment and theories has been found for both following and opposing currents. Both experimental and theoretical results have shown that the spectral densities in the equilibrium range for the waves generated on a following current and an opposing current are larger and smaller, respectively, than those for the waves generated on quiescent water. The influence of current has also been shown to increase with frequency.

In the present study, we have shown that in the equilibrium range when the wind is opposing the flow, the spectral densities tend to be lower than when it follows the flow. This is different from the results of usual wave-current

interaction problems. However, it should be noted that the result of the present study is confined only to the equilibrium range. It is expected that the present study may contribute to the extension of such ocean wave prediction models as WAM (WAMDI group, 1988) or SWAN (Booij et al., 1996) to include current effects.

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APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

- D = quantity defined by (9);
- d = water depth;
- f_a = absolute frequency;
- g = gravitational acceleration;
- h = the highest elevation of wind measurement;
- K = von Karman constant;
- k = wave number;
- L = wave length;
- P = a constant related to turbidity of fluid;
- p = directional parameter used in Gadzhiyev et al. (1978);
- R = universal function defined in Gadzhiyev et al. (1978);
- S = spectral density;
- U = vertically uniform current velocity;
- U_e = equivalent uniform current;
- u = depth-varying horizontal current velocity;
- u_{\max} = maximum horizontal current velocity;
- u_* = current shear velocity;
- v = horizontal wind velocity;
- v_{\max} = maximum horizontal wind velocity;
- v_* = wind shear velocity;
- y = vertical distance measured upward from still water level;
- z = vertical distance measured upward from bed surface;
- α = Phillips' constant;
- β = Phillips-like constant for waves generated on currents;
- γ = peak enhancement factor;
- ε = fraction of wavelength over which the upper part of current profile is averaged in order to establish equivalent uniform current;
- κ = universal function defined in Gadzhiyev et al. (1978);
- σ = spectral width parameter;
- Φ = spectral shape functions;
- ω = angular frequency;
- ω_a = absolute angular frequency;

$$\omega_d = \omega(d/g)^{1/2};$$

ω_r = relative angular frequency;

$$\omega_u = \omega_a U / g .$$

Superscript

e = equilibrium range spectrum.

Subscript

J = JONSWAP;

K = Kitaigordskii et al.;

m = measurement;

o = quantities in quiescent water;

P = Phillips;

p = peak frequency;

S = Suh et al.

Table 1. Test Conditions and Calculated Spectral Parameters

| Depth (cm) (1) | Current ID (2) | u_{max} (cm/s) (3) | u_* (cm/s) (4) | Wind ID (5) | v_{max} (cm/s) (6) | v_* (cm/s) (7) | f_{ap} (Hz) (8) | $\bar{\alpha}$ (9) |
|----------------------|----------------------|----------------------------|------------------------|-------------------|----------------------------|------------------------|-------------------------|-----------------------|
| 50 | NO | 0 | 0 | W1 | 5.71 | 0.262 | 2.009 | 0.0294 |
| | | | | W2 | 8.73 | 0.479 | 1.679 | 0.0460 |
| | | | | W3 | 11.77 | 0.728 | 1.438 | 0.0732 |
| 50 | F1 | 9.43 | 0.301 | W1 | 6.06 | 0.322 | 2.431 | |
| | | | | W2 | 8.91 | 0.521 | 2.005 | |
| | | | | W3 | 11.97 | 0.812 | 1.718 | |
| 50 | F2 | 20.66 | 0.582 | W1 | 5.92 | 0.306 | 3.219 | |
| | | | | W2 | 8.96 | 0.533 | 2.452 | |
| | | | | W3 | 11.93 | 0.807 | 1.995 | |
| 50 | F3 | 30.39 | 0.820 | W1 | 5.98 | 0.250 | 4.078 | |
| | | | | W2 | 8.86 | 0.462 | 2.724 | |
| | | | | W3 | 11.79 | 0.731 | 2.240 | |
| 50 | O1 | 11.48 | 0.392 | W1 | 5.85 | 0.339 | 1.485 | |
| | | | | W2 | 8.89 | 0.642 | 1.198 | |
| | | | | W3 | 11.70 | 0.857 | 1.232 | |
| 50 | O2 | 23.56 | 0.955 | W1 | 5.70 | 0.260 | 1.194 | |
| | | | | W2 | 8.84 | 0.527 | 1.024 | |
| | | | | W3 | 11.68 | 0.774 | 0.943 | |
| 50 | O3 | 34.48 | 1.193 | W1 | 5.67 | 0.271 | 1.012 | |
| | | | | W2 | 8.69 | 0.486 | 1.010 | |
| | | | | W3 | 11.37 | 0.831 | 0.830 | |
| 40 | NO | 0 | 0 | W1 | 5.83 | 0.296 | 1.961 | 0.0299 |
| | | | | W2 | 8.78 | 0.505 | 1.688 | 0.0486 |
| | | | | W3 | 12.11 | 0.871 | 1.459 | 0.0740 |
| 40 | F1 | 9.85 | 0.383 | W1 | 5.84 | 0.274 | 2.537 | |
| | | | | W2 | 8.92 | 0.484 | 2.014 | |
| | | | | W3 | 11.84 | 0.724 | 1.655 | |
| 40 | F2 | 18.86 | 0.522 | W1 | 5.84 | 0.244 | 3.113 | |
| | | | | W2 | 9.06 | 0.464 | 2.336 | |
| | | | | W3 | 12.19 | 0.729 | 1.927 | |
| 40 | F3 | 28.35 | 0.772 | W1 | 5.95 | 0.206 | 4.084 | |
| | | | | W2 | 9.08 | 0.467 | 2.750 | |
| | | | | W3 | 11.98 | 0.648 | 2.159 | |
| 40 | O1 | 11.35 | 0.597 | W1 | 5.80 | 0.260 | 1.473 | |
| | | | | W2 | 8.93 | 0.473 | 1.295 | |
| | | | | W3 | 11.94 | 0.733 | 1.206 | |
| 40 | O2 | 23.18 | 1.022 | W1 | 5.69 | 0.218 | 1.186 | |
| | | | | W2 | 8.69 | 0.411 | 1.013 | |
| | | | | W3 | 11.62 | 0.632 | 0.884 | |
| 40 | O3 | 34.99 | 1.237 | W1 | 5.60 | 0.221 | 1.010 | |
| | | | | W2 | 8.58 | 0.370 | 1.005 | |
| | | | | W3 | 10.99 | 0.472 | 0.763 | |
| 30 | NO | 0 | 0 | W1 | 5.91 | 0.312 | 1.915 | 0.0285 |
| | | | | W2 | 9.21 | 0.605 | 1.607 | 0.0437 |
| 30 | F1 | 10.49 | 0.418 | W1 | 6.17 | 0.322 | 2.449 | |
| | | | | W2 | 9.34 | 0.537 | 1.902 | |
| 30 | F2 | 18.47 | 0.670 | W1 | 6.17 | 0.323 | 2.917 | |
| | | | | W2 | 9.30 | 0.558 | 2.189 | |
| 30 | O1 | 13.03 | 0.657 | W1 | 5.66 | 0.245 | 1.446 | |
| | | | | W2 | 8.89 | 0.473 | 1.182 | |
| 30 | O2 | 25.40 | 1.204 | W1 | 5.80 | 0.226 | 1.170 | |
| | | | | W2 | 8.92 | 0.421 | 0.995 | |

Table 2. Comparison of β/α between Experiment and Theories

| Figure No. (1) | Depth (cm) (2) | Current ID (3) | Wind ID (4) | α (5) | β/α | | |
|-------------------|-------------------|-------------------|----------------|-----------------|-------------------|-------------------------------|-------------------------|
| | | | | | Experiment (6) | Gadzhiyev et al.(1978) (7) | Suh et al.(1994) (8) |
| 5 | 30 | F2 | W1 | 0.0285 | 2.72 | 3.24 | 3.81 |
| 6 | 40 | F2 | W1 | 0.0299 | 2.73 | 3.45 | 4.12 |
| 7 | 50 | F2 | W1 | 0.0294 | 3.01 | 3.77 | 4.63 |
| 8 | 40 | F1 | W1 | 0.0299 | 1.53 | 2.04 | 2.17 |
| 9 | 40 | F3 | W1 | 0.0299 | 5.63 | 5.82 | 8.34 |
| 10 | 40 | F2 | W2 | 0.0486 | 1.74 | 2.84 | 3.22 |
| 11 | 40 | F2 | W3 | 0.0740 | 1.94 | 2.51 | 2.77 |
| 12 | 40 | O1 | W3 | 0.0740 | 0.49 | 0.45 | 0.50 |
| 13 | 50 | O1 | W3 | 0.0732 | 0.48 | 0.44 | 0.50 |

Captions of Figures

1. Plan of Wind-Wave and Current Flume.
2. Illustration of Experimental Setup.
3. Measured and Curve-Fitted Current Profiles: (a) $d = 50$ cm; (b) $d = 40$ cm; (c) $d = 30$ cm.
4. Measured and Curve-Fitted Wind Profile ($d = 50$ cm, No current, Wind = W2).
5. Comparison of β/α between Experiment and Theory at St. E ($d = 30$ cm, Current = F2, Wind = W1).
6. Comparison of β/α between Experiment and Theory at St. E ($d = 40$ cm, Current = F2, Wind = W1).
7. Comparison of β/α between Experiment and Theory at St. E ($d = 50$ cm, Current = F2, Wind = W1).
8. Comparison of β/α between Experiment and Theory at St. E ($d = 40$ cm, Current = F1, Wind = W1).
9. Comparison of β/α between Experiment and Theory at St. E ($d = 40$ cm, Current = F3, Wind = W1).
10. Comparison of β/α between Experiment and Theory at St. E ($d = 40$ cm, Current = F2, Wind = W2).
11. Comparison of β/α between Experiment and Theory at St. E ($d = 40$ cm, Current = F2, Wind = W3).
12. Comparison of β/α between Experiment and Theory at St. E ($d = 40$ cm, Current = O1, Wind = W3).
13. Comparison of β/α between Experiment and Theory at St. E ($d = 50$ cm, Current = O1, Wind = W3).
14. Evolution of Wave Spectra with Fetch ($d = 40$ cm, Current = F3, Wind = W1).